

Other vector spaces

1. Consider the set of all polynomials $\mathbf{R}[x]$. What is the *zero vector* in this vector space? The term zero vector is italicized because the zero vector will be a polynomial.
2. Suppose that the current position at time $t = 0$ of a device along a rail is at 3.54 m, it is moving at 0.32 m/s, and it is currently accelerating at -0.02 m/s^2 . Find a linear combination of 1, x , and x^2 that approximates the current movement of this device.
3. Devices for mixing chemicals often use rotational movement as opposed to back-and-forth shaking, as this reduces the stress on the machine in question. This includes magnetic stirrers and orbital shakers. Suppose you'd like to approximate $3.25 \sin(5.3 + 132.5t)$ by $a_1t + a_0$ by finding evaluating the sinusoid at $t = 0$, differentiating that sinusoid and then evaluating that derivative at $t = 0$, and then taking the appropriate linear combination of 1 and t .
4. Consider the set of all infinite sequences or discrete signals. What is the zero vector in this vector space?
5. Given the vector $x = (1, -0.5, 0.25, -0.125, 0.0625, \dots)$, what vector would need to be added to this vector so that the sum is the zero vector?
6. When Spirit and Opportunity were sent to Mars, their intended lifetime was approximately 92 days. Had their systems been modeled with finite-dimensional vectors spanning across that finite period, the designers may not have been able to detect anomalies to the system that may have occurred after 100, 1000 or 10000 days. Why is it fortunate that usually when engineers design systems, they use infinite sequences instead of finite sequences to model the response of a given system they are designing?
7. Consider the set of all continuous functions or continuous signals $x(t)$ defined for $t \geq 0$. What is the zero vector in this space?
8. Suppose that one function is $9.2e^{-t}\sin(8.3t + 0.1)$ and another is $-5.7e^{-2t}\sin(4.6t + 1.2)$ for $t \geq 0$. If each of these represents the response of two separate systems to a jolt (a shock such as a power surge or lightning strike, or a bump in the road). Suppose that the output of these two systems is being combined (or added). What is the combined response if the first response is first attenuated by 90% (multiplying by 0.1) and the second response is first attenuated by 80% (multiplying by 0.2)?

You should be able to identify with a curve of the form $ae^{-bt}\sin(ct + d)$: imagine you are on a swing, and you stop *pumping* the swing and begin to swing freely. You continue to oscillate back and forth with approximately the same period, but with each swing, the distance you move (or amplitude) decreases. With each period, the change in the amplitude is reasonably small.

Similarly, if your car hits a pot hole, the springs cause car to bounce up and down so as to absorb part of the shock of the pot hole. Unlike a swing, however, a car also has shock absorbers that dampen the oscillations, so that only after at most a few oscillations, the car is steady again. For your entertainment, see what happens if you have no springs on your car: <https://www.youtube.com/watch?v=w1N0QN0zIDY>. Other less entertaining videos show what happens when the shock absorbers are damaged.